

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2019

SECOND YEAR (BATCH 2017-20)

MATHEMATICS (General)

Date : 28.05.2019

Time : 11.00 am-2.00 pm

Paper : IV

Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer any four questions from Question No. 1 to 6 :

[4×5=20]

1. a) Test the convergence of $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$ by Comparison test.

b) Prove that $\int_0^{\infty} e^{-x^2} dx$ converges.

[3+2]

2. a) Test for convergence : $\int_0^1 \frac{dx}{x(x+1)}$.

b) Evaluate $\int_0^{\infty} e^{-x^2} x^3 dx$.

[3+2]

3. Show that the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$.

[5]

4. Find the length of the curve $ay^2 = x^3$ between the points $x=0$ and $x=5a$.

[5]

5. Find the volume of the solid generated by revolving the curve $r = 1 - \cos \theta$ about the initial line.

[5]

6. Find the area of the surface formed by the revolution of the curve $x^2 + 4y^2 = 16$ about its major axis.

[5]

Answer any two questions from Question No. 7 to 9 :

[2×5=10]

7. Solve : $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$.

[5]

8. Solve : $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = xe^x - e^x + 2e^{3x}$

[5]

9. Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$, where 'a' is a variable parameter.

[5]

Group - B

Answer any three questions from Question No. 10 to 14 :

[3×15=45]

10. a) Discuss about primary and secondary data and their sources.

[2+2+1]

b) Following are the heads of income of Indian Railway Corporation in the year 2015 and 2016 :

	2015 (in Crore of Rs.)	2016 (in Crore of Rs.)
Coaching	26	31
Goods	40	39
Others	4.50	3.50

Represent the above data by a bar diagram and analyse your views.

[4+1]

c) The expenditure of 1000 families is given as following

Expenditure (in Rs.)	40-59	60-79	80-99	100-119	120-139
No. of families	50	?	500	?	50

The median and mean for the distribution are both 87.50. Calculate the missing frequencies.

[5]

11. a) Find the Mean Deviation about the Arithmetic Mean of the following series :

[5]

x	10	11	12	13	14
Frequency	3	12	18	12	3

b) Two variables x and y are related by $y=10-3x$. If the Standard Deviation of x is 4, what will be the Standard Deviation of y?

[5]

c) Find the coefficient of Correlation from the following data :

[5]

X	1	2	3	4	5	6	7
Y	6	8	11	9	12	10	14

12. a) Construct the price index Number of 2015 with 2000 as Base year from the following data :

[5]

Commodity	Price (Rs./kg)		Quantity Sold (Kg.)	
	2000	2015	2000	2015
A	4	5	95	120
B	60	70	118	130
C	35	40	50	70

b) Determine the equation of a straight line which best fits the following data :

Year	2001	2002	2003	2004	2005
Sales (in '00000 Rs.)	35	56	79	80	40

Compute the trend values for all the years from 2001 to 2005.

[3+3]

c) The joint probability distribution of a pair (X,Y) of random variables is given in the following table :

Y \ X	X	1	2	3
	Y			
1		0.1	0.1	0.2
2		0.2	0.3	0.1

Find the conditional distribution of Y , given X = 2.

[4]

13. a) If the letters of the word RAMESH be arranged at random, what is the probability that there are exactly 3 letters between A and E. [5]

b) If events A^c and B^c are independent, show that so are A and B. [3]

c) Find the expectation and variance of binomial (n,p). [3+4]

14. a) If X and Y are two jointly distributed continuous random variables and also they are independent, then prove $E(XY)=E(X) \cdot E(Y)$. [3]

b) If minimum daily temperature in a certain city follows a normal distribution with a mean of 59°F and a standard deviation of 9°F , then what would be the distribution of minimum daily temperature in Centigrade scale? Hence find the probability that the minimum daily

temperature of a particular day will be more than 20°C . [Given $\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.3413$]. [3+4]

c) If (X_1, X_2, \dots, X_n) be a random sample from a normal population $N(\theta, 1)$, then show that

$\left(\overline{X}^2 - \frac{1}{n} \right)$ is an unbiased estimator of θ^2 . [5]

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